MARKING SCHEME — MATHEMATICS CLASS XII (80 MARKS)

SECTION A (Q.1 – Q.20 : 1 mark each)

Q.1 (C) reflexive and transitive but not symmetric **1**

Q.2 (A) one-one and onto **1**

Q.3 (B) $-\frac{\pi}{6}$ **1**

Q.4 (B) $[-1, 1]$ **1**

Q.5 (B) $-\frac{\pi}{2}$ $[\frac{\pi}{3} - \frac{5\pi}{6}]$ **1**

Q.6 (C) 3×2 **1**

Q.7 (B) 3 **1**

Q.8 (B) a symmetric matrix **1**

Q.9 (C) continuous at $x = 0$ but not differentiable at $x = 0$ **1**

Q.10 (B) $\cot x$ **1**

Q.11 (B) $4e^{2x}$ **1**

Q.12 (B) $(0, \infty)$ **1**

Q.13 (C) $(-1, 1)$ $[f'(x) = 3x^2 - 3 < 0$ for $-1 < x < 1]$ **1**

Q.14 (B) 20π cm²/s $[\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(5)(2)]$ **1**

Q.15 (A) $\tan x + C$ **1**

Q.16 (B) 1 **1**

Q.17 (B) 2 sq units **1**

Q.18 (C) 9 sq units **1**

Q.19 (A) Both true, (R) correctly explains (A) **1**

Q.20 (A) Both true, (R) correctly explains (A) **1**

SECTION B (Q.21 – Q.25 : 2 marks each)

Q.21 $\tan^{-1}(1) = \frac{\pi}{4}$, $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$, $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ 1 **2**

Sum = $\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi + 8\pi - 2\pi}{12} = \frac{3\pi}{4}$ 1

OR

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ 1

Value = $\frac{2\pi}{3} + 2 \times \frac{\pi}{6} = \pi$ 1

Q.22 $A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, so $A + A' = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$ 1½ **2**

$(A + A')' = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} = A + A'$; hence it is symmetric ..... ½

Q.23 Put $t = x^2 + 1$, so $dt = 2x \, dx$ 1 **2**

$\int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \log|x^2 + 1| + C$ 1

Q.24 The highest order derivative is $\frac{d^2y}{dx^2}$, so the order is 2 ..... 1 **2**

The power of this highest order derivative is 3, so the degree is 3 1

Q.25 Separating the variables : $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$ 1 **2**

Integrating : $\tan^{-1}y = \tan^{-1}x + C$ 1

OR

$\frac{dy}{dx} = e^x \cdot e^y \Rightarrow e^{-y} dy = e^x dx$ 1

Integrating : $-e^{-y} = e^x + C$, i.e. $e^x + e^{-y} = C$ 1

SECTION C (Q.26 – Q.31 : 3 marks each)

Q.26 Reflexive : 3 divides $(a - a) = 0$ for all $a \in Z$ ..... ½ **3**

Symmetric : if 3 divides $(a - b)$ then 3 divides $(b - a)$ ..... ½

Transitive : if 3 divides $(a - b)$ and $(b - c)$, then 3 divides their sum $(a - c)$ 1

Hence R is an equivalence relation ..... ½

Equivalence classes : $[0], [1], [2]$ — the integers leaving remainders 0, 1, 2 on division by 3 ..... ½

Q.27 (a) $sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ 1 **3**

(b) $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ 1

(c) Domain of $tan^{-1}x$ is R ; principal value branch is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ 1

OR

(a) $cot^{-1}(-1) = \frac{3\pi}{4}$ 1

(b) Range (principal value branch) of $cos^{-1}x$ is $[0, \pi]$ 1

(c) $\frac{2\pi}{3} \in [0, \pi]$, so $cos^{-1}\left(cos\,\frac{2\pi}{3}\right) = \frac{2\pi}{3}$ 1

Q.28 $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \Rightarrow A = -1 , B = 2$ 1½ **3**

$\int \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx$ ..... ½

$= -\log|x + 1| + 2\log|x + 2| + C$ 1

Q.29 Using integration by parts with $u = x$ and $dv = sin\,x\;dx$ 1 **3**

$\int x\,sin\,x\;dx = -x\,cos\,x + \int cos\,x\;dx$ 1

$= -x\,cos\,x + sin\,x + C$ 1

Q.30 Required area $= 2\int_0^4 2\sqrt{x}\,dx$ (by symmetry about the x-axis) 1 **3**

$= 4 \times \frac{2}{3}\left[x^{3/2}\right]_0^4 = \frac{8}{3} \times 8 = \frac{64}{3}$ sq units 2

Q.31 Area $= \int_0^4 \sqrt{16 - x^2}\,dx$ 1 **3**

$= \left[\frac{x}{2}\sqrt{16 - x^2} + 8\,sin^{-1}\left(\frac{x}{4}\right)\right]_0^4 = 8 \times \frac{\pi}{2} = 4\pi$ sq units 2

OR

Area $= 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\,dx$ 1

$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 = \frac{8}{3} \times \frac{9\pi}{4} = 6\pi$ sq units 2

SECTION D (Q.32 – Q.35 : 5 marks each)

Q.32 (a) Reflexive : $|a - a| = 0$ is divisible by 2 ; Symmetric : $|a - b| = |b - a|$ 1 **5**
Transitive : if $|a - b|$ and $|b - c|$ are even then a, b, c have the same parity, so $|a - c|$ is even 1
Hence R is an equivalence relation ; classes are $[1] = \{1, 3, 5\}$ and $[2] = \{2, 4\}$ 1
(b) $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$ (as $x \in N$) ; hence one-one 1
2 ∈ N has no pre-image in N since $\sqrt{2} \notin N$; hence f is not onto 1

Q.33 (a) $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$ ..... ½ **5**
$P = \frac{1}{2}(A + A') = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$ — symmetric 1
$Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ — skew-symmetric ; $A = P + Q$ 1
(b) $AB = \begin{bmatrix} 4 & 6 \\ 10 & 12 \end{bmatrix}$ 1
$(AB)' = \begin{bmatrix} 4 & 10 \\ 6 & 12 \end{bmatrix}$ ..... ½
$B'A' = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 6 & 12 \end{bmatrix} = (AB)'$; hence verified 1

Q.34 (a) $\log y = \sin x \cdot \log x$ 1 **5**
$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \log x + \frac{\sin x}{x}$ 1
$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \log x + \frac{\sin x}{x}\right)$ ..... ½
(b) $\frac{dx}{d\theta} = a(1 - \cos\theta)$, $\frac{dy}{d\theta} = a\sin\theta$ 1
$\frac{dy}{dx} = \frac{\sin\theta}{1 - \cos\theta}$; at $\theta = \frac{\pi}{2}$, $\frac{dy}{dx} = 1$ 1½

OR

LHL = $\lim_{x \to 1^-}(3x - 2) = 1$; RHL = $\lim_{x \to 1^+}(2x^2 - x) = 1 = f(1)$; so f is continuous at $x = 1$ 2
Left derivative = 3 ; Right derivative = $(4x - 1)$ at $x = 1 = 3$ 2
Since the two derivatives are equal, f is differentiable at $x = 1$ 1

Q.35 (a) $f'(x) = 6x^2 - 18x + 12 = 6(x - 1)(x - 2)$ 1 **5**
Strictly increasing on $(-\infty, 1) \cup (2, \infty)$ 1
Strictly decreasing on $(1, 2)$ 1
(b) $A = \pi r^2$, so $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ 1
$= 2\pi(10)(3) = 60\pi$ cm²/s 1

OR

(a) $f'(x) = 3x^2 - 6x + 4 = 3(x - 1)^2 + 1 > 0$ for all $x \in R$ 2
Hence f is strictly increasing on R 1
(b) $V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 9 = 3(100)\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{3}{100}$ 1
$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x\frac{dx}{dt} = 12(10)\left(\frac{3}{100}\right) = 3.6$ cm²/s 1

SECTION E (Q.36 – Q.38 : case study, 4 marks each)

Q.36 (i) $f(2) = 200 + 50(2) = 300$ 1 **4**
(ii) The claim means f is continuous at $x = 2$, i.e. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$ 1
(iii) $2k + 100 = 300 \Rightarrow k = 100$ 1
Bill for 5 GB = $100(5) + 100 = ₹\,600$ 1

OR

(iii) With $k = 120$: RHL $= 120(2) + 100 = 340 \neq 300 = f(2)$, so f is not continuous at $x = 2$ 1

Jump in the bill $= 340 - 300 =$ ₹ 40 1

Q.37 (i) $V = \frac{4}{3}\pi r^3$ 1 **4**

(ii) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ 1

(iii) $900 = 4\pi(225)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi}$ cm/s 2

OR

(iii) $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi(15)\left(\frac{1}{\pi}\right) = 120$ cm²/s 2

Q.38 (i) $\frac{dy}{dt} = ky$, where k is the constant of proportionality 1 **4**

(ii) Order 1 and degree 1 1

(iii) $\frac{dy}{y} = k, dt \Rightarrow \log y = kt + C \Rightarrow y = y_0 e^{kt}$ with $y_0 = 1000$ 1

$4000 = 1000\,e^{2k} \Rightarrow e^{2k} = 4$; after 4 hours $y = 1000\left(e^{2k}\right)^2 = 1000 \times 16 = 16000$ bacteria 1

OR

(iii) $y = 1000\,e^{kt}$ and $e^{2k} = 4$, i.e. $k = \frac{1}{2}\log 4$ 1

$16000 = 1000\,e^{kt} \Rightarrow e^{kt} = 16 = (e^{2k})^2 \Rightarrow t = 4$ hours 1

Note : Competency-based / application questions (Q.9, 12, 13, 14, 17, 18, 19, 20, 24, 30, 31, 33, 35, 36–38) ≈ 42 marks (53%), meeting the CBSE requirement of at least 50% competency-focused assessment.