BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY |CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

MID-TERM EXAMINATION 2023-24

APPLIED MATHEMATICS (241)

Class : XII Commerce Duration: 3 Hrs.
Date : 18-10-2023 MARKING SCHEME Max. Marks: 80

Question	Answer	Scheme
1	A is a matrix of order 2x3 and B is a matrix of order 3x2. Product AB is defined and Product BA is also defined. Order (AB)=2x2. Order (BA)=3x3	Answer C
2	It satisfies the condition that elements equidistant from leading diagonal are equal in value but opposite in sign. Therefore, it is a skew symmetric matrix	Answer C
3	In general, product of two matrices are not commutative. Therefore, -AB and BA cannot get cancelled.	Answer C
4	Product of AB both of order 3x3 should be calculated. Now $C(1,3)$ entry $= y + 0 + x$. It is compared with zero as per the identity matrix of order 3x3.	Answer A
5	Compare both determinants; $3x^2 - 20 = -8 - (-15);\ 3x^2 = 7 + 20;\ x^2 = 9$ Therefore, $x = 3$	Answer C
6	Determinant of the matrix = 0 as it is given singular. $\lvert A\rvert = 1(9x - 6x^2) - 3(9 - 4x^2) + 9(6 - 4x) = 9x - 6x^2 - 27 + 12x^2 + 54 - 36x = 0$ =$6x^2 - 27x + 27 = 0\ or\ (2x - 3)(x - 3) = 0\ or\ x = \frac{3}{2}\ or\ 3$	Answer C
7	Determinant of A = a^3 . But $\lvert adj\ A\rvert = \lvert A\rvert^2$. Therefore, $\lvert adj\ A\rvert = \lvert A\rvert^2 = (a^3)^2 = a^6$	Answer C
8	It can be simplified as $y^2 = x + y$. Differentiate the equation implicitly we get, $2y \times \frac{dy}{dx} = 1 + \frac{dy}{dx}$. When simplified we get, $\frac{dy}{dx} = \frac{1}{2y-1}$	Answer C
9	Parametric form: $x = t^2\ becomes \frac{dx}{dt} = 2t$ $y = t^3\ becomes \frac{dy}{dt} = 3t^2$. Now $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{2t} = \frac{3}{2}t$ Now $\frac{d^2y}{dx^2} = \frac{3}{2} \times \frac{dt}{dx} = \frac{3}{2} \times \frac{1}{2t} = \frac{3}{4t}$	Answer D
10	Given $x^2 + xy + y^2 = 0$. Differentiate with respect to x; $2x + x\frac{dy}{dx} + y(1) + 2y\frac{dy}{dx} = 0$ or $(x + 2y)\left(\frac{dy}{dx}\right) = -(2x + y)\ or \frac{dy}{dx} = -\frac{2x+y}{x+2y}$	Answer B
11	Let $u = log\ x$ and $v = \frac{1}{x}$ Now $\frac{du}{dx} = \frac{1}{x}\ and \frac{dv}{dx} = -\frac{1}{x^2}$ Further $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \frac{1}{x} \times \left(-\frac{x^2}{1}\right) = -x$	Answer C
12	Equation of the curve $y = e^{2x}$. Now $\frac{dy}{dx} = e^{2x} \times 2$. Slope of the tangent at (0, 1) is $\frac{dy}{dx} at\ (0,1) = e^{2(0)} \times 2 = 2$ Equation of the tangent is $y - 1 = 2(x - 0)\ or\ y - 1 = 2x$	Answer C

13	Polynomial $f(x) = x^3 – 18x^2 + 96x$ and hence $f'(x) = 3x^2 - 36x + 96$ Now $f'(x) = 0\ implies\ x^2 - 12x + 32 = 0\ or\ (x - 8)(x - 4) = 0.$ Stationary points are $x = 4\ and\ x = 8.$ The smallest value of the polynomial is 0. It happens at 0 given in $[0, 9]$	Answer D
14	Cost function $C(x) = 30x + 240$ and Revenue function $R(x) = 45x.$ Breakeven point is obtained when $R(x) = C(x)$ Now $45x = 30x + 240$. Therefore, $15x = 240$. Hence $x = 16$	Answer D
15	Population is denoted by the symbol μ. FALSE Population is not a statistic. FALSE.	Answer D
16	If $p\ value \leq \alpha$ then Reject Ho. It means reject null hypothesis.	Answer A
17	A fire in a factory delaying production for some time, happens unexpected, very rarely.	Answer D
18	3 monthly moving averages calculated from 5 data would have 3 entries.	Answer B
19	Assertion: False Revenue function $R(x) = px = 200x - \frac{x^3}{3}.$ Marginal Revenue $MR = \frac{dR}{dx} = 200 - x^2; MR\ at\ x = 10\ is\ 200 - 10\text{^}2 = Rs. 100$ Reason: True	Answer D
20	Assertion: True Reason: True But reason is not in support of the Assertion statement.	Answer B

21	Comparing entries in two equal matrices, we have $3y + 1 = 16\ or\ 3y = 15\ or\ y = 5$ Take $x + y = 8$. We come to know $x = 8 - y = 8 - 5 = 3$ Now $y - x = 5 - 3 = 2$ is the answer	1 0.5 0.5
22	A is a 3x3 matrix. Given $\|A\| = 4$ Now $\|2A\| = 2^3\|A\| = 8 \times 4 = 32$ is the answer	1 1
23	Apply log on both sides, then $log\ y = y\ log\ x.$ Differentiate with respect to x, we get $\frac{1}{y} \times \frac{dy}{dx} = y \times \frac{1}{x} + \log x \times \frac{dy}{dx}$ When simplified $\frac{dy}{dx} = \frac{\frac{y}{x}}{\frac{1}{y} - \log x} = \frac{y^2}{x(1 - y \log x)}$	0.5 1 0.5
OR	Given $y = x^3 \log x$ then $\frac{dy}{dx} = x^3 \times \frac{1}{x} + \log x \times 3x^2 = x^2 + 3x^2 \log x$ Now $\frac{d^2y}{dx^2} = 2x + 3x^2 \times \frac{1}{x} + \log x \times 6x = 5x + 6x \log x$ is the answer	1 1
24	Number of items $n = 50$, population standard deviation $\sigma = 6$. Sample mean $\bar{x} = 32$ Confidence level = 95%. It means $1 - \alpha = 0.95; \alpha = 0.05; \frac{\alpha}{2} = 0.025$ Therefore, $Z_{\alpha/2} = Z_{0.025} = 1.96$ from area table of values Now Margin of Error = $Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{6}{\sqrt{50}} = 1.96 \times 0.848 = 1.66$ So, $\mu = \bar{x} \pm margin\ of\ error = 32 \pm 1.66$ Confidence interval = $(32 - 1.66, 32 + 1.66) = (30.34, 33.66)$	0.5 0.5 0.5 0.5
25	Cost function $C(x) = 24000 + 2x$ and Revenue function $R(x) = 8x.$ To find breakeven point we take $R(x) = C(x)$ then $8x = 24000 + 2x$; We get $6x = 24000$ and so $x = 4000\ Rs.$	1 1

26	Eqn(1)x2 , $4x-2y=\begin{bmatrix}12&-12&0\\-8&4&2\end{bmatrix}$ Eqn(2), $x+2y=\begin{bmatrix}3&2&5\\-2&1&-7\end{bmatrix}$ Adding both equations, $5x=\begin{bmatrix}15&-10&5\\-10&5&-5\end{bmatrix}$ Therefore, $x=\begin{bmatrix}3&-2&1\\-2&1&-1\end{bmatrix}$ Now $y=2x-\begin{bmatrix}6&-6&0\\-4&2&1\end{bmatrix}=\begin{bmatrix}6&-4&2\\-4&2&-2\end{bmatrix}-\begin{bmatrix}6&-6&0\\-4&2&1\end{bmatrix}=\begin{bmatrix}0&2&2\\0&0&-3\end{bmatrix}$	0.5 0.5 0.5 0.5 1
27	Apply transformations $R_2=R_2-4R_1\ and\ R_3=R_3-8R_1$ Now $\begin{vmatrix}x+y&x&x\\x&0&-2x\\2x&0&-5x\end{vmatrix}=-x(-5x^2+4x^2)=x^3$	1 1 1
OR	Coefficients D = $\begin{vmatrix}1&1&1\\1&2&3\\1&3&4\end{vmatrix}=1(8-9)-1(4-3)+1(3-2)=-1-1+1=-1$ D1 = $\begin{vmatrix}-1&1&1\\-4&2&3\\-6&3&4\end{vmatrix}=-[1(8-9)-1(16-18)+1(12-12)]=-[-1+2]=-1$ D2 = $\begin{vmatrix}1&-1&1\\1&-4&3\\1&-6&4\end{vmatrix}=-[1(16-18)-1(4-3)+1(6-4)]=-1[-2-1+2]=1$ D3 = $\begin{vmatrix}1&1&-1\\1&2&-4\\1&3&-6\end{vmatrix}=-[1(12-12)-1(4-3)+1(6-4)]=-[-1+2]=-1$ Therefore, $x=\frac{D_1}{D}=\frac{-1}{-1}=1;y=\frac{D_2}{D}=\frac{1}{-1}=-1;z=\frac{D_3}{D}=\frac{-1}{-1}=1$	0.5 0.5 0.5 0.5 0.5 0.5
28	Differentiating we get $2ax+2h\left(x\times\frac{dy}{dx}+y\times1\right)+2b\times\frac{dy}{dx}+2g+2f\times\frac{dy}{dx}+0=0$ So, $(2hx+2by+2f)\frac{dy}{dx}=-2ax-2hy-2g$ Dividing by 2, $\frac{dy}{dx}=-\left(\frac{ax+hy+g}{hx+by+f}\right)$	1 1 1
29	Volume of cylindrical container =$V=\pi r^2h$ Differentiating with respect to time, $\frac{dv}{dt}=\pi r^2\left(\frac{dh}{dt}\right);314=\pi\times10^2\left(\frac{dh}{dt}\right)$ Therefore, $\frac{dh}{dt}=\frac{314}{3.14\times100}=1\ m/hr$	1 1 1
OR	Cost of production $C(x)=300x^2+4200x+13500$ Revenue function $R(x)=8400x$. Breakeven point is when $R(x)=C(x)$ Therefore, $8400x=300x^2+4200x+13500;300x^2-4200x+13500=0$ Simplified form is $x^2-14x+45=0;(x-9)(x-5)=0;\therefore x=9\ or\ x=5$	1 1 1
30	Differentiating we get $a2y\times\frac{dy}{dx}=3x^2;\frac{dy}{dx}=\frac{3x^2}{2ay}$ Slope of the tangent at $(am^2,am^3)is=m=\frac{3a^2m^4}{2a^2m^3}=\frac{3m}{2}$ Equation of the tangent is $y-am^3=\frac{3m}{2}(x-am^2)$ That is, $2y-2am^3=3mx-3am^3;3mx-2y=am^3$ is the equation of tangent	1 1 1
31	Test statistic $t=\frac{\bar{x}-\mu}{s/\sqrt{n}}=\frac{103-100}{11.5}\times\sqrt{65}=\frac{3\times8.062}{11.5}=\frac{24.186}{11.5}=2.10>0$ Degrees of freedom = $n-1=64$ So, p-value of 2.10 = 2 x area under t-distribution curve to the right of t.From t-distribution table, we find $t=2.10$ lies between 1.998 and 2.386 for which area lies between 0.010 and	1 1 1

<table>
<tr><td></td><td>0.025, so p-values lies between $2x0.010\ and\ 2x0.025$. that is, 0.020 and 0.050.
Since$0.020 < p - value < 0.050$, Reject null hypothesis.</td><td></td></tr>
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<tr><td>32</td><td>Population proportion $\bar{p} = \frac{281}{611} = 0.4599$
Margin of Error = $Z_{\frac{\alpha}{2}} \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = Z_{0.05} \times \sqrt{\frac{0.45\times0.54}{611}} = 1.645 \times \sqrt{\frac{0.2484}{611}} = 1.645 \times \sqrt{0.00040}$
=$1.645 \times 0.02015 = 0.0332$
At 90% confidence interval, $\alpha = 0.1\ and\ hence \frac{\alpha}{2} = 0.05$
Interval is $(\bar{p} \pm ME) = (0.4267{,}0.4931)$</td><td>1
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<tr><td>33</td><td>
<table>
<tr><th>Year</th><th>y</th><th>$x = t - 2004$</th><th>x^2</th><th>xy</th></tr>
<tr><td>2001</td><td>30</td><td>-3</td><td>9</td><td>-90</td></tr>
<tr><td>2002</td><td>35</td><td>-2</td><td>4</td><td>-70</td></tr>
<tr><td>2003</td><td>36</td><td>-1</td><td>1</td><td>-36</td></tr>
<tr><td>2004</td><td>32</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>2005</td><td>37</td><td>1</td><td>1</td><td>37</td></tr>
<tr><td>2006</td><td>40</td><td>2</td><td>4</td><td>80</td></tr>
<tr><td>2007</td><td>36</td><td>3</td><td>9</td><td>108</td></tr>
<tr><td>Total</td><td>$\sum y = 246$</td><td>$\sum x = 0$</td><td>$\sum x^2 = 28$</td><td>$\sum xy = 29$</td></tr>
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$a = \frac{\sum y}{n} = \frac{246}{7} = 35.14\ and\ b = \frac{\sum xy}{\sum x^2} = \frac{29}{28} = 1.036$
The required equation of the trend line is $y = 35.14 + 1.036x$
Trend value for the year 2008 is $y = 35.14 + 1.036 \times (2008 - 2004) = 39.284$</td><td>1
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1</td></tr>
<tr><td>34</td><td>Given $c(x) = \frac{x^3}{3} + 3x^2 - 7x + 16$
Marginal cost function is $\frac{dc}{dx} = x^2 + 6x - 7$
Average cost function is $AC\ = \frac{C(x)}{x} = \frac{x^2}{3} + 3x - 7 + \frac{16}{x}$
Marginal average cost MAC =$\frac{dAC}{dx} = \frac{2x}{3} + 3 - \frac{16}{x^2}$
$\frac{x(MC) - c(x)}{x^2} = \frac{x^3 + 6x^2 - 7x - \frac{x^3}{3} - 3x^2 + 7x - 16}{x^2} = \frac{2x}{3} + 3 - \frac{16}{x^2}$</td><td>1
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1</td></tr>
<tr><td>OR</td><td>Volume of the open box = $V = (18 - 2x)(18 - 2x)x = 324x - 72x^2 + 4x^3$
$\frac{dv}{dx} = 324 - 144x + 12x^2 = 12(x^2 - 12x + 27)$
If $\frac{dv}{dx} = 0, then\ (x - 9)(x - 3) = 0$. Therefore, v is maximum when $x\ =\ 3$
Maximum volume = $(18 – 2 \times 3)(18 - 2 \times 3)3 = 12 \times 12 \times 3 = 432\ sq.cm$</td><td>1
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1
1
1</td></tr>
<tr><td>35</td><td>First derivative $\frac{dy}{dx} = x^3 \times \frac{1}{\frac{1}{x}} \times \frac{-1}{x^2} + \log\left(\frac{1}{x}\right) \times 3x^2 = -x^2 + 3x^2 log\left(\frac{1}{x}\right)$
$\frac{dy}{dx} = -x^2 + 3x^2 \log 1 - 3x^2 \log x = -x^2 - 3x^2 \log x$
$\frac{d^2y}{dx^2} = -2x - 3x^2 \times \frac{1}{x} - \log x \times 6x = -5x - 6x \log x$
Now $xy_2 - 2y_1 + 3x^2 = x(-5x - 6x \log x) - 2(-x^2 - 3x^2 \log x) + 3x^2$
$= -5x^2 - 6x^2 \log x + 2x^2 + 6x^2 \log\ x + 3x^2 = 0$</td><td>1
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OR	Let $u = x^{\sin x}\ then \log u = \sin x \times x$ Differentiating we get $\frac{1}{u} \times \frac{du}{dx} = \sin x \times 1 + x \times \cos x$ Therefore , $\frac{du}{dx} = x^{\sin x}(\sin x + x \cos x)$ Let $v = \log x^x\ \ then \log v = x \times \log x$ Differentiating we get $\frac{1}{v} \times \frac{dv}{dx} = x \times \frac{1}{x} + \log x \times 1$ Therefore, $\frac{dv}{dx} = \log x^x\ (1 + \log x)$ Finally, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\sin x}(\sin x + x \log x) + \log x^x\ (1 + \log x)$	1 1 1 1 1

36a	Now $\frac{dy}{dx} = (x+2)(1-0) + (x-3)(1+0) = x + 2 + x - 3 = 2x - 1$	1
36b	Here $x \times 2y\left(\frac{dy}{dx}\right) + y^2 \times 1 = 0\ or \frac{dy}{dx} = -\frac{y^2}{2xy} = -\frac{y}{2x}$	1
36c	Differentiating, $\frac{dy}{dx} = \frac{(6-4x)(3)-(3x-2)(-4)}{(6-4x)^2} = \frac{18-12x+12x-8}{(6-4x)^2} = \frac{10}{(6-4x)^2}$	1 1
OR	Now $2x + 2y \times \frac{dy}{dx} = 0;\ or\ \frac{dy}{dx} = -\frac{x}{y}$	1 1
37a	Equations formed are $\frac{10x}{100} + \frac{12y}{100} = 2800\ and \frac{12x}{100} + \frac{10y}{100} = 2700$ In simplified form: $5x\ +\ 6y\ =\ 140000\ and\ 6x\ +\ 5y\ =\ 135000$	0.5 0.5
37b	Product $A^2 = \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} = \begin{bmatrix}25 & 0\\0 & 25\end{bmatrix}$ We are able to learn that $A^3 = \begin{bmatrix}125 & 0\\0 & 125\end{bmatrix}$	0.5 0.5
37c	Adding both equations we get $11x + 11y = 275000;\ or\ x + y = 25000$ Subtracting them we get $-x + y = 5000$ These equations eventually added gives $2y = 30000\ or\ y = 15000\ Rs.$ From equation (1) we get $x = Rs. 10000$. These are the two investments in bonds.	1 1
OR	In this context $A = \begin{bmatrix}5 & 6\\6 & 5\end{bmatrix}$ is the matrix of coefficients. Now $adj\ A = \begin{bmatrix}5 & -6\\-6 & 5\end{bmatrix}. Further\ \vert adj\ A\vert = 25 - 36 = -11(= \vert A\vert)$	1 1
38a	New monthly charge = $Rs. 300\ +\ x$ Total revenue = $R\ =\ (300\ +\ x)(500 - x)$	0.5 0.5
38b	Now $\frac{dR}{dx} = (300 + x)(-1) + (500 - x)(1) = -300 - x + 500 - x = 200 - 2x$ Critical point is $\frac{dR}{dx} = 0\ implies\ 200 - 2x = 0\ and\ hence\ x = 100$ At $x\ =\ 100$, revenue will be maximum	0.5 0.5
38c	Revenue is maximum when the subscription is increased by $Rs. 100$ Therefore, number of subscribers left = $500 - 100\ =\ 400$	1 1
OR	New charges per subscriber = $Rs. 300\ +\ Rs. 100\ =\ Rs. 400$ and The number of subscribers left = 400 Maximum revenue generated = $Rs. 400\ x\ 400\ =\ Rs. 160000$	1 1